1

Unit

Rotational Dynamics and Oscillatory Motion

Johannes Kepler
(1517 – 1630)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the concept the rotation of rigid bodies.
- Explain the terms of moment of inertia, moment of force (torque) of rotating body.
- Describe the angular momentum and its conservation principle.
- Know about the rotational kinetic energy.
- Understand the concept of oscillatory motion and simple harmonic motion.
- Describe the characteristics of simple harmonic motion (SHM).
- Describe about the oscillating spring.
- Calculate the total energy of the oscillating body (conservation of total energy).
- Solve the various numerical problems related with rotational dynamics and oscillatory motion.

8. Distinguish between periodic motion and oscillatory motion.

Ans: The motion, which repeats itself after equal interval of time, is called a periodic motion. If a body moves back and forth (to and fro) repeatedly about a mean position, then it is said to possess oscillatory motion. For example, the revolution of a planet around the sun is a periodic motion but not an oscillatory motion. Therefore, all oscillatory motion are periodic but all periodic motion may not necessarily to be oscillatory.

9. A clock based on an oscillatory spring is taken to the moon. What will be the time period there?

Ans: For an oscillatory spring, time period T is $T = 2\pi\sqrt{\frac{m}{K}}$. Where, m is the mass attached and K is the spring constant. Here, both m and K are constants. For an oscillatory spring, the time period T is independent with the value of g. Therefore, the time period T remains same on the surface of the moon as it is on earth's surface.

10. To double the total energy for an oscillatory mass-spring system, by what factor must the amplitude increase? What effect does this change have on the frequency?

Ans: Total energy E of simple harmonic motion is, $E = \frac{1}{2}m\omega^2r^2$ ... (1)

If r_1 be the new amplitude for double energy, then $2E = \frac{1}{2}m\omega^2r_1^2$ ...(2)

Dividing equation (2) by equation (1), we get

$$\frac{2E}{E} = \frac{r_1^2}{r^2} \Rightarrow r_1^2 = 2r^2 \Rightarrow r_1 = \sqrt{2}\,r$$

So, to double energy, amplitude should increase by $\sqrt{2}$ times.

Also, frequency f is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{... (3)}$$

This shows that f is independent with the amplitude r of motion. Hence frequency f remains constant.

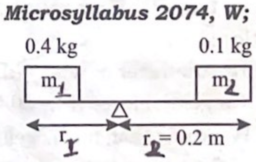
11. A body is moving in circular path with constant speed. Is this motion a simple harmonic? Why?

Ans: The motion of a body in a circular path with constant speed is a periodic motion. The magnitude of acceleration is constant and the body never comes at rest. The motion is not oscillatory also. But, in SHM, the acceleration should be directly proportional to displacement and, its value is maximum at extreme position and zero at mean position. Hence, the motion of a body in circular path with constant speed is not simple harmonic.

Worked Out Examples

1. A balance scale consisting of a weightless rod has a mass of 0.1 kg on the right side 0.2 m from a pivot point. (a) How far from the pivot point on the left must 0.4 kg be placed so that balance is achieved? (b) If the 0.4 kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) What is the instantaneous tangential acceleration of the 0.1 kg mass when the 0.4 kg mass is removed?

[TU Microsyllabus 2074, W; 8.1]



[Fig. 17: Masses from pivot point]

Solution:

Mass of a balance scale (m) = 0.1 kg

Distance from the pivot point (r_2) = 0.2 m

Distance from the pivot point of mass 0.4 kg is r_1 = ?

Instantaneous rotational acceleration (Torque) α = ?

Instantaneous tangential acceleration (a) = ?

a. The torque on both sides must be equal for achieving a balance. So that, $\tau_1 = \tau_2$ (principle of moment)

$$r_1F_1 \sin 90^\circ = r_2F_2 \sin 90^\circ$$

$$r_1m_1g = r_2m_2g$$

$$r_1 = \frac{r_2m_2}{m_1} = \frac{0.2 \times 0.1}{0.4} = 0.05 \text{ m}$$

Hence, at 0.05 m distance from the mass 0.4kg, balance is achieved.

b. We know that, Torque is given by

$$\tau = I\alpha$$

or, $\alpha = \frac{\tau}{I} = \frac{r_2F_2 \sin 90^\circ}{m_2r_2^2}$

or, $\alpha = \frac{F_2 \sin 90^\circ}{m_2r_2}$

or, $\alpha = \frac{m_2g}{m_2r_2} = \frac{g}{r_2} = \frac{9.8}{0.2} = 49 \text{ rad/sec}^2$.

Thus, rotational acceleration (Torque) $\alpha = 49 \text{ rad/sec}^2$

c. We have the relation, $a = r\alpha = 0.2 \times 49 = 9.8 \text{ m/sec}^2$

Hence, Instantaneous tangential acceleration $a = 9.8 \text{m/sec}^2$

2. A large wheel of radius 0.4 m and moment of inertia 1.2 kgm², pivoted at the centre is free to rotate without friction. A rope is wound it and a 2 kg weight is attached to the rope. When the weight has descended 1.5 m from its starting point (a) what is its downward velocity? (b) What is the rotational velocity of wheel? ***[TU Microsyllabus 2074, W; 8.2]***

Solution:

The radius of the wheel (r) = 0.4 m

Moment of inertia (I) = 1.2 kgm²

Weight (m) = 2 kg

Height (h) = 1.5 m

Downward velocity (V) = ?

Rotational velocity (ω) = ?

a. We know that, from the conservation of energy

Potential energy of weight = Kinetic energy of weight + Rotational kinetic energy of wheel.

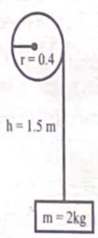
$\therefore$ $mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \frac{v^2}{r^2}$ Where, $\omega = \frac{v}{r}$

$2mgh = \left(m + \frac{I}{r^2}\right) v^2$

or, $v = \sqrt{\frac{2mgh}{\left(m + \frac{I}{r^2}\right)}} = \sqrt{\frac{2 \times 2 \times 9.8 \times 1.5}{\left(2 + \frac{1.2}{0.4^2}\right)}}$

$\therefore$ $v = 2.5 \text{ m/sec}$.



[Fig. 18: Weight attached to the rope]

b. We have the relation,

$\therefore$ $\omega = \frac{v}{r} = \frac{2.5}{0.4} = 6.2 \text{ rad/sec}$.

Hence, required downward velocity and rotational velocity are 2.5m/sec and 6.2 rad/sec respectively.

3. A machine shop a lathe wheel of 40 cm diameter driven by belt that goes around the rim. If the linear speed of the belt is 2m/sec and the wheel requires a tangential force of 50N to turn it, how much power is required to operate the lathe?

Solution:

The diameter of wheel (d) = 2r = 40 cm = 0.4 m

Linear speed (v) = 2 m/sec

Tangential force (F) = 50 N

Power required to operate the Lathe (P) = ?

We know that, linear velocity (v) = $r\omega$

or, $\omega = \frac{v}{r} = \frac{2}{0.2} = 10 \text{ rad/sec}$.

Again, the torque (τ) = rF Sin ϕ = 0.2 × 50 × Sin 90° = 10 Nm

Then, power (P) = $\omega\tau$ = 10 rad/sec × 10 Nm

= 100 W = 0.13 Hp   Since, 746 W = 1 Hp

Hence, required power to operate the lathe = 0.13 Hp.

4. Suppose the body of an ice skater has a moment of inertia 4 kgm² and her arms have a mass 5 kg each with the centre of mass at 0.4m from her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms out stretched. She then pulls her arms inward so that their centre of mass is at the axis of her body is zero. What is the speed of rotation? ***[TU Microsyllabus 2074, W; 8.4]***

Solution:

Moment of inertia of a body (I) = 4 kgm²

Mass of body arms (m) = 5 kg

Distance (r) = 0.4 m

Angular frequency (ω) = 0.5 rev/sec.

Speed of rotation ω_f = ?

We know that,

$$I_0 \omega_0 = I_f \omega_f$$

$$(I_{body} + I_{arms}) w_0 = I_{body}\, \omega_f$$

$$(I_b + 2mr^2)\, \omega_0 = I_b\, \omega_f$$

$$\omega_f = \frac{(I_b + 2mr^2)\,\omega_0}{I_b} = \frac{4kgm^2 + 2 \times 5\ kg \times (0.4m)^2}{4\ kgm^2} \times 0.5\ rev/sec$$

$\therefore$ ω_f = 0.7 rev/sec.

Hence, required speed of rotation ω_f = 0.7rev/sec.

5. A given spring stretches 0.1m when a force of 20N pulls on it. A 2 kg block attached to it on a frictionless surface pulled to the right 0.2 m and released.

a. What is the frequency of oscillation of the block?

b. What is its velocity at the midpoint?

c. What is its acceleration at either end?

d. What are the velocity and acceleration when x = 0.12 m on the block's first passing this point? ***[TU Microsyllabus 2074, W; 10.2]***

Solution:

The spring stretches (x) = 0.1 m

Force (F) = 20N

Mass of block (m) = 2 kg

Distance toward right (r) = 0.2m

Frequency of oscillation (f) = ?

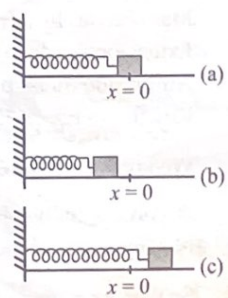
Velocity at the mid point (V_{max}) = ?

Velocity and acceleration (a_{max}) = ?

Velocity and acceleration when x = 0.12m on the blocks first passing = ?

We have, F = kx

or, $k = \frac{F}{x} = \frac{20}{0.1} = 200\ N/m$



[Fig. 19: Stretched string at different positions]

a. We know that frequency (f) = $\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200}{2}}$ = 1.6 Hz

b. Maximum velocity (V_{max}) = rω = 0.2 × 2π × 1.6 = 2 m/sec

c. Maximum acceleration (a_{max}) = rω^2 = 0.2 × (2π × 1.6)² = 20.19 m/sec

d. Velocity when x = 0.12 m passing this point,

$$V = \omega\sqrt{r^2 - x^2} = 2\pi \times 1.6 \times \sqrt{0.2^2 - 0.12^2} = 1.6\ m/sec$$

and $a = \omega^2 x = (2\pi \times 1.6)^2 \times 0.12 = 12.12\ m/sec^2$.

6. A spring mass system consists of a 2 kg mass block and spring constant 200 N/m. The block is released from the position of x_1 = 0.2m.

a. What is its velocity at x_2 = 0.1 m?

b. What is the acceleration at this point? *[TU Microsyllabus 2074, W; 10.3]*

Solution:

Mass of the block (m) = 2 kg

Velocity of block when x_2 = 0.1 m is = ?

Spring constant (K) = 200 N/m

Acceleration of block (a) = ?

a. We have from the conservation of energy

$$\frac{1}{2}kx_1^2 + \frac{1}{2}m v_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

$$kx_1^2 + mv_1^2 = kx_2^2 + m_2v_2^2.$$

If $v_1 = 0$ i.e., start from rest position.

So, that $kx_1^2 = kx_2^2 + m_2v_2^2$

$$\therefore \quad v_2 = \sqrt{\frac{k(x_1^2 - x_2^2)}{m}} = \sqrt{\frac{200\,(0.2^2 - 0.1^2)}{2}} = 1.73\text{m/sec}$$

Thus, v_2 = 1.73 m/sec

b. We know that, F = ma

kx = ma

Since, F = kx for simple harmonic motion

$$\therefore \quad a = \frac{kx}{m} = \frac{200 \times 0.1}{2} = 10\text{m/sec}^2$$

Hence, velocity and acceleration are 1.73 m/sec and 10 m/sec^2 respectively.

7. A Linear spring whose force constant is 0.2N/m hangs vertically supporting a 1 Kg mass at rest. The mass is pulled down a distance 0.2 m and then released. What will be its maximum velocity? Also find the frequency of vibration.

Solution:

The spring constants (k) = 0.2 N/m

Distance (Amplitude) (r) = 0.2 m

Mass (m) = 1 kg

Maximum velocity and frequency of vibration = ?

$$\text{We have } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.2}{1}} = 0.447$$

Maximum velocity $V_{max} = r\omega = 0.2 \times 0.447 = 0.089$m/sec

$$\text{The frequency } f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times 0.447 = 0.071\text{Hz}$$

Thus, the required maximum velocity and frequency are 0.089 m/sec and 0.071 Hz respectively.

8. A light spring is suspended from a rigid support and its free and carries a mass of 0.4 kg, which produces an extension of 6 cm in the string. The mass is then pulled down further 4 cm and then released so that the mass oscillate with simple harmonic motion, Calculate the kinetic energy generated as it passed through mean position.

Solution:

Mass carries by a free end of string (m) = 0.4 kg.

Extension produce (x) = 6 cm = 0.06 m

Amplitude (mass pull down distance) (r) = 40cm = 0.04m

Kinetic energy (K.E.) = ?

We know that, maximum kinetic energy $(KE)_{max} = \frac{1}{2}kr^2$

Since, r ≈ v (distance covered)

According to the SHM, F = kx and

Newton's law of motion, F = ma.

$$\text{So that, } kx = ma \Rightarrow k = \frac{ma}{x} = \frac{0.4 \times 9.8}{0.06} = 65.33\text{n/m}$$

$$\text{The } (K.E)_{max} = \frac{1}{2}(65.33) \times (0.04)^2 = 0.0523\text{J}$$

Thus, the required maximum kinetic energy is 0.0523J.

9. A bicycle wheel of mass 2 kg and radius 0.32 m is spinning freely on its axle at 2 rev sec^{-1}. When you place your hand against the tyre, the wheel decelerates uniformly and comes to a stop in 8 sec. What was the torque of your hand against the wheel?

[TU Microsyllabus 2074, P; 8.1]

Solution

Mass of wheel (m) = 2 kg

Radius of wheel (r) = 0.32 m

Initial spinning frequency (f_0) = 2 rev sec^{-1}

Time taken (t) = 8 sec

Torque (τ) = ?

From given condition,

Initial angular frequency (ω_0) = $2\pi f_0$

Final angular frequency (ω_f) = $2\pi f_f = 0$

We know that

$\tau = I\alpha$

Where, I = moment of inertia,

α = angular acceleration

Then,

$$\tau = mr^2\left(\frac{\omega_f - \omega_0}{t}\right) \qquad \text{Since, } \alpha = \frac{\omega_f - \omega_0}{t}$$

$$= 2 \times (0.32)^2\left(\frac{2\pi f_f - 2\pi f_0}{t}\right)$$

$$= 2 \times (0.32)^2 \times \left(-\frac{2\pi f_0}{t}\right) \qquad \text{Since, } f_f = 0$$

$$= 2 \times (0.32)^2 \times \left(-\frac{2\pi \times 2}{8}\right)$$

$$\therefore \tau = -0.64 \text{ Nm}$$

Hence, the torque of our hand against the wheel is –0.64 Nm.

10. A grindstone with I = 240 kgm^2 rotates with a speed of 1 rev sec^{-1}. A knife blade is pressed against it, and the wheel coasts to a stop with constant declaration in 12 sec. What torque did the knife exert on the wheel? ***[TU Microsyllabus 2074, P; 8.4]***

Solution:

Here is given, moment of inertia of grindstone (I) = 240 kgm^2

Initial speed of revolution per second (f_0) = 1 rev sec^{-1}

Time taken up to stop (t) = 12 sec

Torque (τ) = ?

We know that,

$$\tau = I\alpha$$

$$= I\left(\frac{\omega_f - \omega_0}{t}\right)$$

$$= \frac{240 \times (0 - 2\pi \times 1)}{12}$$

$$\therefore \quad \tau = 125.66 \text{ Nm}$$

Hence, the required torque exerted by knife on the wheel is 125.66 Nm.

11. Two masses, m_1 = 1kg and m_2 = 5 kg are connected by a rigid rod of negligible weight. The system is pivoted about point O. The gravitational forces act in the negative direction

a. Express the position vectors and the forces on the masses in terms of unit vectors and calculate the torque on the system.

b. What is the angular acceleration of the system at the instant shown in figure 20 below?

[TU Microsyllabus 2074, P; 8.2]

Solution:

Here is given,

a. Two masses

m_1 = 1 kg

m_2= 5 kg

Here,

Position vector can be expressed as;

$\vec{r_1} = -2\,\hat{j}$ m

$\vec{r_2} = 4\,\hat{j}$ m

Then

$\vec{F_1} = -10\,\hat{k}$ N Since, F = mg and gravitational forces acts in the negative z-direction.

$\vec{F_2} = -50\,\hat{k}$ N

Now torque on the system

$\vec{\tau} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$

$= -2\hat{j} \times (-10\,\hat{k}) + 4\,\hat{j} \times (-50\,\hat{k})$

$= 20\hat{i} - 200\,\hat{i}$

$= (20 - 200)\,\hat{i}$ Nm

$\therefore$ $\vec{\tau} = -180\,\hat{i}$ Nm

b. We know that,

Torque $(\vec{\tau}) = I\vec{\alpha}$

or, $\tau = I\alpha$

or, $\alpha = \frac{\tau}{I}$

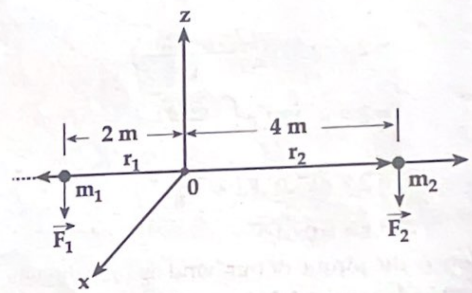
$= \frac{\tau}{(I_1 + I_2)}$

$= \frac{\tau}{(m_1r_1^2 + m_2r_2^2)}$

$= \frac{\tau}{1 \times (-2)^2 + 5 \times 4^2}$

$= \frac{-180\,\hat{i}}{84}$

$\therefore$ $\vec{\alpha} = -2.14\,\hat{i}$ rad s^{-1}

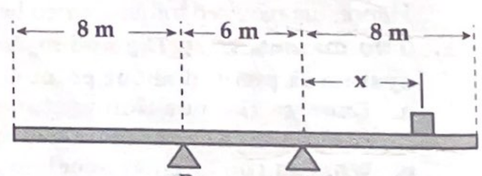


[Fig 20: Representation of two masses connected by a rigid rod of negligible weight]

Hence, required torque and angular acceleration of the system are $-180\,\hat{i}$ Nm and $-2.14\,\hat{i}$ rad s^{-1} respectively.

12. A uniform wooden board of mass 20 kg rests on two supports as shown in figure. A 30 kg steel block is placed to the light of support A. How far to the right of a can the steel the block be placed without tipping the board? ***[TU Microsyllabus 2074, P; 8.7]***

Solution:



[Fig 21: A uniform wooden board rests on two rigid supports]

Here is given,

Mass of uniform wooden board

$M_1 = M_w = 20$ kg

Mass of steel block $M_2 = M_s = 30$ kg

According to question distance between two supports is 6m, so centre of gravity (C.G.) of uniform wooden board lies between A and B as shown in figure 21. Let 'x' be the right of A at which steel block is placed without tipping the board. Then,

$r_1 = 3$m

$r_2 = x$ m

Now applying principle of conservation of moment

Sum of clockwise moment = Sum of anticlockwise moment

$M_2gr_2 = M_1gr_1$

$30\times10\times x = 20\times10\times3$

Therefore, x = 2m

Hence, steel block must be placed at 2m right from A.

13. A children's merry go-round of radius 4 m and mass 100 kg has an 80 kg man standing at the rim. The merry-go-round coasts on a frictionless bearing at 0.2 rev s^{-1}. The man walks inward 2 m toward the centre. What is the new rotational speed of the merry-go-round? What is the source of this energy? (The moment of inertia of a solid disk is $I = \frac{1}{2} mr^2$)

[TU Microsyllabus 2074, P; 8.18]

Solution:

Here is given, radius of merry go round $(r_1) = 4$ m

Mass of merry go round (M) = 100 kg

Mass of man (m) = 80 kg

Frequency (f) = 0.2 rev s^{-1}

Now for rotational speed (f_2), According to conservation angular momentum and energy

$I_1\omega_1 = I_2\omega_2$

According to question moment of inertia of solid disk $I = \frac{1}{2}mr^2$. Then

$I_1 = (I + I'_1)$ and $I_2 = (I + I'_2)$

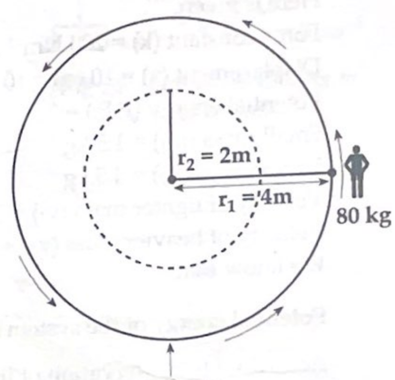
or, $\left(\frac{1}{2}mr_1^2 + Mr_1^2\right) 2\pi f_1 = \left(\frac{1}{2}mr_1^2 + Mr_2^2\right) 2\pi f_2$

or, $\left(\frac{1}{2}100 \times 4^2 + 80 \times 4^2\right) \times 0.2 = \left(\frac{1}{2}100 \times 4^2 + 80 \times 2^2\right) \times f_2$

or, $416 = 1120\ f_2$

$\therefore$ $f_2 = 0.371$ rev s^{-1}

Hence, required new rotational speed of the merry go round is 0.371 rev s^{-1}.



[Fig 22: A children's merry-go-round]

14. An oscillating block of mass 250 g takes 0.15 s to move between the end points of the motion, which are 40 cm apart. (a) What is the frequency of the motion? (b) What is the amplitude of the motion? (c) What is the force constant of the spring?

[TU Microsyllabus 2074, P; 10.5]

Solution:

Here is given, block of mass (m) = 250 gm = 250×10^{-3} kg

Time (t) = 0.15 sec

Displacement (x) = 40 cm = 40×10^{-2} m

Frequency (f) = ?

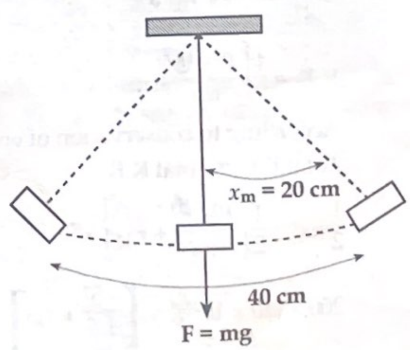
Amplitude of a motion (x_m) = ?

Force constant (k) = ?

We know that,

a. $f = \frac{1}{t} = \frac{1}{0.15} = 6.67$ rev s^{-1}.

b. Amplitude $(x_m) = \frac{40 \times 10^{-2}}{2} = 20 \times 10^{-2}$m



[Fig 23: Oscillating block between two end points]

c. Again we know that

$$2\pi f = \sqrt{\frac{k}{m}}$$

or, $4\pi^2 f^2 = \frac{k}{m}$

or, $k = 4\pi^2 f^2 m$

or, $k = 4\pi^2 (6.67)^2 \times 250 \times 10^{-3}$

$\therefore$ $k = 439.08\ \text{Nm}^{-1}$

Hence, required frequency, amplitude and force constant are 6.67 rev s^{-1}, 20×10^{-2} m and 439.08 Nm^{-1} respectively.

15. A spring ($k = 200\ \text{Nm}^{-1}$) is compressed 10 cm between two blocks of mass $m_1 = 1.5$ kg and $m_2 = 4.5$ kg. The spring is not connected to the blocks and the table is frictionless. What are the velocities of the blocks after they are released and lose contact with the spring? Assume that the spring falls straight down to the table. ***[TU Microsyllabus 2074, P; 10.18]***

Solution:

Here is given,

Force constant (k) = 200 Nm^{-1}

Displacement (x) = 10 cm = 10×10^{-2} m

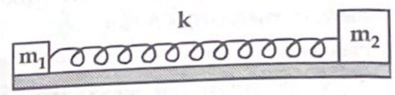
Potential energy (P.E.) = ?

Small mass (m_1) = 1.5 kg

Large mass (m_2) = 4.5 kg

Velocity of lighter mass (v_1) = ?

Velocity of heavier mass (v_2) = ?



[Fig 24: A spring connected with two blocks of having different masses]

We know that,

Potential energy of the system (P.E.) $= \frac{1}{2} kx^2$ . . . (1)

According to conservation of linear momentum

$m_1 v_1 + m_2 v_2 = 0$

$m_1 v_1 = - m_2 v_2$

$v_1 = \frac{-m_2}{m_1} v_2$ . . . (2)

Now kinetic energy of the system

$$(\text{K.E.}) = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$$

$$= \frac{1}{2}\left[m_1 \left(\frac{-m_2}{m_1} v_2\right)^2 + m_2 v_2^2\right]$$

$$= \frac{1}{2}\left[\frac{m_2^2 v_2^2}{m_1} + m_2 v_2^2\right]$$

$\therefore$ $\text{K.E.} = \frac{1}{2}\left[\frac{m_2^2}{m_1} + m_2\right] v_2^2$ . . . (3)

According to conservation of energy

Total P.E. = Total K.E.

$$\frac{1}{2} kx^2 = \frac{1}{2}\left[\frac{m_2^2}{m_1} + m_2\right] v_2^2$$

$$200 \times (10 \times 10^{-2})^2 = \left[\frac{4.5^2}{1.5} + 4.5\right] v_2^2$$

$$v_2^2 = \frac{200 \times (10 \times 10^{-2})^2}{18}$$

$\therefore$ $v_2 = 3.33 \times 10^{-1} \simeq 0.33\ \text{ms}^{-1}$

Then, using value of v_2 in equation (2), we get

$$v_1 = -\frac{4.5}{1.5} \times 0.33$$

$$\therefore \quad v_1 = -1 \text{ ms}^{-1}$$

Hence, velocities of masses m_1 and m_2 after they are released are -1 ms^{-1} and 0.33 ms^{-1} respectively.

16. A block is oscillating with amplitude of 20 cm. The spring constant is 150 Nm^{-1}. (a) What is the energy of the system? (b) When the displacement is 5 cm, what are the kinetic energy of the block and the potential energy of the spring? ***[TU Microsyllabus 2074, P; 10.13]***

Solution:

Amplitude (x_m) = 20 cm = 20×10^{-2} m

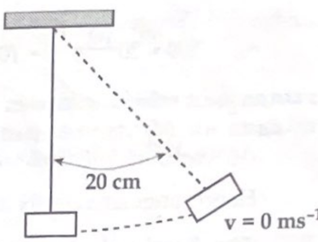
Spring constant (k) = 150 Nm^{-1}

Energy of the system (E) = ?

Displacement (x) = 5 cm = 5×10^{-2} m

Kinetic Energy (K.E.) = ?

Potential Energy (P.E.) = ?



[Fig 25: A block oscillating in the vertical plane]

a. Total energy (T.E.) = Kinetic energy (K.E.) + Potential energy (P.E.)

$$= \frac{1}{2} K x_m^2 \quad \text{Since, } v = 0 \text{ at maximum amplitude}$$

Then, $$= \frac{1}{2} 150 \times (20 \times 10^{-2})^2$$

$$= 3.0 \text{ J}$$

b. $$\text{K.E.} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} k(x_m^2 - x^2)$$

$$= \frac{1}{2} 150 [(20 \times 10^{-2})^2 - (5 \times 10^{-2})^2]$$

$$= 2.81 \text{ J}$$

c, $$\text{P.E.} = \frac{1}{2} kx^2$$

$$= \frac{1}{2} \times 150 \times (5 \times 10^{-2})^2$$

$$\therefore \text{P.E.} = 0.19 \text{ J}$$

Hence, required energy of the system, kinetic energy at 5 cm apart and potential energy of the spring are 3.0 J, 2.81 J and 0.19 J respectively.

Additional Numerical Examples

1. A ballet dancer spins with 2.4 rev/sec with her arms outstretched when the moment of inertia about the axis of rotation is I. With her arms folded, the moment of inertia about the same axis becomes 0.6I. Calculate the new rate of spin.

Solution:

Given, Initial frequency (f_1) = 2.4 rev/sec

Initial moment of inertia (I_1) = I

Final moment of inertia (I_2) = 0.6 I

Final frequency (f_2) = ?

From angular momentum conservations principle,

$$I_2\omega_2 = I_1\omega_1$$

or, $$I_2 \times 2\pi f_2 = I_1 \times 2\pi f_1$$

$$\therefore \quad f_2 = \frac{I_1 \times f_1}{I_2} = \frac{I \times 2.4}{0.6\, I} = 4 \text{ rev/sec.}$$